

The effects of alternative power-sharing arrangements: Do “moderating” institutions moderate party strategies and government policy outputs?

Samuel Merrill III · James Adams

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Abstract Advocates of consensual political institutions, i.e. institutions that promote compromise and powersharing among political parties, claim that these institutions promote moderation in government policy outputs. To date, however, there exists little research – either theoretical or empirical – that evaluates whether consensual institutions promote moderation in parties’ policy declarations. We develop a multiparty spatial model with policy-seeking parties operating under proportional representation, in which we vary the extent to which government policies reflect power-sharing among all parties as opposed to being determined by a single party. We determine parties’ optimal (Nash equilibrium) policy positions and conclude that power-sharing does not typically motivate parties to moderate their policy declarations; in fact, policy positioning under power-sharing appears to be similar to or more extreme than under single-party dominance. Consistent with previous research, however, we find that power-sharing does promote moderation in government policy outputs. Our results have implications for parties’ election strategies, for the design of political institutions, and for representative government.

Keywords Spatial model · Party strategy · Formateur · Power-sharing · Nash equilibrium

1 Introduction

An electorate’s capacity to choose a representative government depends on the policy positions presented by competing parties during the election campaign as well as on implemented policies after election. Although campaign statements are readily available to the voters, after election they may not be faithfully reflected in government policy. The extent to which

S. Merrill III (✉)

Department of Mathematics and Computer Science, Wilkes University

Present address: 3024 43rd Ct. NW, Olympia, WA 98502, USA

e-mail: smerrill@zhonka.net

J. Adams

Department of Political Science, University of California, Davis, CA 95616, USA

implemented policy deviates from parties' pre-election policy declarations, however, can only be surmised by the voters. Thus voter decisions depend significantly on pre-election policy declarations.

Is there reason to expect that either party declarations or government outputs vary with institutional arrangements? Under proportional representation systems, it would be surprising if outputs did not depend on whether government is dominated by consensus among all parliamentary parties, by a coalition of governing parties, or by a single party. But the effect of such governmental arrangements on parties' pre-election party declarations – which are chosen in a delicate balance between a desire to maintain or increase vote share while advocating desired policy – is far from clear. Shedding light on this latter question is the primary objective of this paper.

Empirical researchers have analyzed the connection between parties' policy declarations and government policy outputs. This wide-ranging research agenda is concerned with cataloguing the institutional arrangements that affect power-sharing in political systems (Lijphart, 1984, 1999), and also with determining how institutional arrangements translate parties' policy preferences into policy outputs (see Huber & Powell, 1994; Powell, 2000; Lijphart, 1999, Chapter 16; McDonald, Mendes, & Budge, 2004; McDonald & Budge, 2006).

A second, more theoretical, line of research involves spatial modeling. Since the work of Downs (1957) the assumption that parties are vote-maximizers has dominated political theorizing. The spatial models that incorporate this assumption, however, do not typically predict the wide divergence of party strategies that is observed empirically. Perhaps the most widely cited explanation for such policy divergence is that the competing parties have policy motivations – i.e., that party elites seek office in order to implement desired policies, rather than proposing policies in order to win office (see Wittman, 1977, 1983; Calvert, 1985; Londregan & Romer, 1993; Groseclose, 2001; Roemer, 2001; Adams, Merrill, & Grofman, 2005; Smirnov & Fowler, forthcoming). To date, however, spatial modeling research on policy-seeking parties and empirical research on political institutions and government policy outputs have evolved largely independently of each other (but see Andrews & Money, 2006).

Our purpose here is to merge these strands of research, by incorporating empirical researchers' conclusions on the determinants of government policy outputs into a spatial model of policy-seeking parties in multiparty elections under proportional representation. Specifically, we take as our starting point empirical findings suggesting that government policy outputs depend critically on the identity of a *dominant party* and on the location of the *parliamentary mean* (i.e., the mean position of all parties in parliament, weighted by seat share), but that the relative policy influence of these variables depends critically upon institutional arrangements that vary across polities. Our model includes parameters that permit us to vary the relative influence upon government policy outputs of a dominant party versus the parliamentary mean. We use this model to analyze both policy-seeking parties' policy strategies, and the government policy outputs that can be expected to result from these strategies.

Our central conclusion is that – depending on assumptions – parties' optimal policy-seeking strategies for the parliamentary-mean model are typically either quite similar to or more extreme than those of the dominant-party model. For both models of policy outputs, parties with noncentrist policy preferences must balance their desire to present positions that reflect these preferences but that attract only modest electoral support – which translates into diminished expectations about post-election policy influence – against the strategy of moderating their policy positions to increase support and expected influence, but at the cost of making policy compromises. We show that under *expressive* voting – i.e. when voters support the party whose policy programme best reflects their policy preferences – this balance of centrifugal and centripetal incentives is typically quite similar – and is in some

cases identical – across these contrasting models of policy outputs. However, when voters are *instrumental* in the sense that they account for the fact that power-sharing arrangements will dilute any single party's parliamentary influence, then voters – and hence parties – take more extreme stands in an effort to pull the parliamentary mean their way. We also present qualitative reasons why our conclusions plausibly extend to an alternative model of policy outputs that accounts for the coalition of parties that forms the government.

We believe the above results have important empirical and normative implications. Empirically, our findings are relevant to an emerging empirical literature that examines how alternative power-sharing arrangements influence parties' tendencies to advocate moderate as opposed to extreme policies (Ezrow, 2005; Budge & McDonald, 2006). These empirical studies do not find a systematic relationship between power-sharing and the extremity of party policy declarations observed in real world party systems – empirical results that our theoretical results illuminate, since we conclude that alternative power-sharing arrangements do *not* consistently affect party extremism, provided that voters are expressive. We note that the locations of party policy declarations – as opposed to policy outputs – are important in and of themselves, since they affect voter expectations and hence voter choice and a sense that voters with diverse viewpoints are represented (see Abney, Morrison, and Stradiotto, 2006; Andrews & Money, 2006). As Cox notes, the representation process is defined “in terms of whether voters can find a legislator who advocates similar views” (1997: 36). Consequently, while it is impossible to implement policies that reflect every citizen's viewpoint, members of the mass public plausibly derive satisfaction when a political party articulates their positions, even if most positions are not enacted into law.

Normatively, our findings are relevant to the longstanding debate over the relative merits of *consensual* democracies, i.e., those that feature institutions that promote power-sharing and compromise, versus *majoritarian* institutions that promote single-party dominance (see Lijphart, 1984, 1999; Powell, 2000; McDonald, Mendes, & Budge, 2004; McDonald & Budge, 2006).¹ Advocates of consensual political institutions typically argue that, all other factors being equal, these institutions promote more moderate policy outputs (see Section 5 below for a literature review). However, to the extent that consensual institutions motivate parties to present different policy declarations than do majoritarian institutions, all other factors are *not* equal. Our results – that alternative power-sharing arrangements do not systematically affect declared party positions when voters choose expressively – thereby supports a key tenet underpinning consensual theory. In this regard, we note that our analyses also support the expectation that post-election power-sharing does indeed moderate the policy *outputs* that can be expected to result from parties' policy declarations.

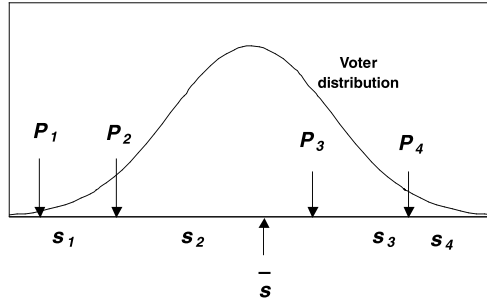
2 The parliamentary-mean model

Unlike plurality systems in which a single party frequently wins a parliamentary majority and thereby dominates the policy-making process,² under PR multiple parties typically exercise

¹ This debate is often framed as a choice between consensual institutions that promote power-sharing, versus majoritarian institutions that promote single-party dominance (see Lijphart, 1984, 1999). We emphasize that while our findings are relevant to this debate – in that we vary the extent of power-sharing in our model by varying the policy influence of the dominant-party versus the parliamentary mean – an *unvarying* feature of our model is that seats are awarded via a PR formula. Given that majoritarian democracies are typically associated with the plurality voting system, our model does not capture the full range of differences between majoritarian and consensual democracies.

² For instance, in both postwar Britain and in New Zealand (prior to the switch to PR in 1996), the plurality party won a parliamentary majority in 15 of 17 postwar elections.

Fig. 1 Policy positions for alternative models of government formation (Party policy positions in a four-party election are denoted by $s_1, s_2, s_3,$ and s_4 . The parliamentary mean is denoted by \bar{s} . The symbols $P_1, P_2, P_3,$ and P_4 indicate the preferred position of the parties)



or have the potential to exercise significant policy influence. We begin with two pure models for the nature of government policy outputs. These pure models are not presented as faithful models of reality per se, but rather their primary significance is as components of more nuanced models that reflect multiple influences on government outputs.

For simplicity, we will assume that seat share is exactly proportional to vote share, i.e., that the PR system is perfectly proportional.³ Suppose there are K parties, with preferred positions P_1, \dots, P_K on a policy scale. We seek Nash equilibrium values for the declared positions s_1, \dots, s_K of the respective parties on the same policy scale (see Figure 1). For simplicity, we restrict our development to a one-dimensional, deterministic voting model and assume that each party's utility for a policy position is based on linear loss, i.e., party k 's utility for position s_j is $-|P_k - s_j|$.

At one extreme, we may suppose that policy on each issue is a compromise among the policy positions of *all* the parties making up the parliament, without regard to whether these parties are in government or opposition – weighted by their seat shares. We term this the *parliamentary-mean* expectation of governmental outcomes. Although our major purpose in introducing the *parliamentary-mean* model is as a component of a more complex model to be considered in the next section, in its pure form it appears most relevant to consensual democracies such as Switzerland, Belgium, & the Netherlands, in which extensive provisions for opposition parties' prerogatives provide these parties a degree of policy influence that may approach that of the governing parties. Warwick (2001) finds empirical support for the parliamentary mean – which he labels the “parliamentary center of gravity” – as a significant predictor of government policy declarations in empirical analyses in ten democracies.⁴

In the parliamentary-mean model, in the view of, say, party k with preferred policy position P_k , party deviations to the left of P_k may be balanced by other parties' deviations to the right of P_k . Accordingly, under compromise expectations, the utility U_k of the outcome to party k depends on the distance from P_k to a weighted average of the party positions, so that the

³ The degree to which real world PR systems approach perfect proportionality depends on several factors, notably *district magnitude* (i.e. the number of seats awarded per district) and the existence (or absence) of *electoral thresholds* defined in terms of a minimum percentage of the national vote a party must win in order to guarantee parliamentary representation (see Taagepera & Shugart, 1989; Cox, 1997; Lijphart, 1999). Among the most perfectly proportional systems are those of Israel, the Netherlands, and the Scandinavian countries (see Lijphart, 1999, Appendix A).

⁴ Specifically, Warwick uses the location of the parliamentary mean and concludes that this mean significantly influences policy outputs, even when accounting for the positions of the governing parties (which typically includes the median party). To measure party policy declarations, Warwick (2001) relies on the codings of party manifestos and government declarations carried out by the Comparative Manifestos Project (see Laver & Budge, 1992; Budge et al., 2001).

utility for party k under the parliamentary-mean model is given by

$$U_k = - \left| P_k - \sum_{j=1}^K s_j EV_j \right| = - |P_k - \bar{s}|, \quad (1)$$

where $\bar{s} = \sum_{j=1}^K s_j EV_j$ is the mean policy of the parties weighted by their expected vote shares, EV_j (see Figure 1).

The parliamentary-mean model does not necessarily induce parties to take extreme stands, because to do so seriously erodes their electoral support so that their influence on the parliamentary mean diminishes toward zero. In fact, parties must balance this erosion of support with their policy objectives, leading typically – as we will see – to center-left and center-right equilibrium positions.

Although we focus on the parliamentary mean as a model of shared or consensual government, an alternative to the parliamentary-mean model is a *parliamentary-median* model, in which government output is the median position in parliament, i.e., the declared position of the median party in parliament, which in turn is that party that captures the support of the median voter. Plausibly one can argue that such a party is in a swing position permitting it to be decisive in parliamentary decisions. Assuming a parliamentary-median model and certainty about the location of the median voter, an equilibrium in party positions occurs as long as two or more of the parties locate at the position of the median voter. In such an equilibrium, other parties can change positions without any change in their utility. Unlike the equilibrium results reported later in this paper for the parliamentary-mean model, this result for a parliamentary-median model is knife-edge because parties – if they have even a slight motivation for office as well as policy – have an incentive to all locate at the median. On the other hand, under a parliamentary-median model, parties diverge if there is uncertainty about the location of the median voter. In a separate paper (Adams & Merrill, 2006), we investigate the consequences of a model in which the median party in parliament determines policy, given uncertainty about the election outcome.

3 The dominant-party model

Alternatively, we may assume that following the election one of the parties becomes primarily responsible for forming policy and that, a priori, the identity of this party is probabilistic, but related to the electoral strength of the parties. We term this assumption the *dominant-party model*. A dominant party may appear, for example, if one party is chosen to be the *formateur* – i.e., the party awarded the opportunity to attempt to form a government – and, at the extreme, this bargaining advantage could allow a dominant party to fully implement its preferred policies. Warwick (2001) reports empirical analyses of government policy declarations from ten Western European political systems, suggesting that the formateur does indeed derive policy advantages.⁵

For the dominant-party model, we further assume that before the election the probability that each party becomes the dominant-party is proportional to its expected seat share, which is in turn equivalent to vote share given our assumption of a perfectly proportional PR

⁵ Specifically, when reparametrized, the coefficients in Warwick's regression suggest that government policy declarations are shaded in the direction of the formateur's policy positions, to an extent that is disproportionate to the formateur's seat share in parliament or to its share of seats in the governing coalition.

electoral system. Diermeier and Merlo (2004) find empirical support for the proportionality proposition in situations where no party receives a majority of seats in parliament, while Baron and Ferejohn (1989) employ this assumption in their formal model of government formation.⁶ There is extensive empirical evidence, furthermore, that government ministries are distributed approximately proportionally to the governing parties' seat shares (Gamson, 1961; Browne & Franklin, 1973). Although there is empirical evidence for the proportionality hypothesis, it is plausible that parties located near the center of the voter distribution have a disproportionate likelihood of becoming dominant. If parties nearer the center of the voter distribution are disproportionately weighted, the effect is similar to proportionate weights for a voter distribution with smaller variance, so that in turn the variance of the configuration of party positions at equilibrium is reduced, as we will see later in Section 7 on numerical calculations.

We note that while our dominant-party model is motivated by the formateur's pre-eminent policy-making position, there are alternative considerations that also suggest a similar model of policy outputs. Consider, for instance, the portfolio allocation theory developed by Laver and Shepsle (1996), which takes as its starting point the observation that when a governmental coalition is formed, one individual representing one party is normally chosen to head the ministry responsible for each policy area. Laver and Shepsle (1996: 31) observe "... a very strongly departmental character to government decision making," because "... government departments are the only organizations with the resources to generate fully developed policy proposals and the expertise to implement and monitor any proposal that might be selected." In assigning each minister, therefore, choice of a particular politician implicitly cedes the power to shape the agenda under the jurisdiction of that ministry to that individual's party. To the extent that the left-right dimension is related to the jurisdiction of the finance minister, Warwick (2001) observes that the government's left-right policy position should be related to that of the party holding the finance ministry. In this sense, a model based upon government portfolio allocations may be linked to the dominant-party model.

Under the dominant-party assumption, given linear loss utilities for the parties, the expected utility U_k of the outcome to party k is the negative of the sum of the distances from the preferred position of party k to the declared positions of the respective parties, weighted by the probability that each respective party will determine government policy. We assume that these latter weights are proportional to the expected vote shares/seat shares of the parties. Thus, the utility for party k under the dominant-party model is

$$U_k = - \sum_{j=1}^K |P_k - s_j| EV_j, \quad (2)$$

where EV_j denotes the expected vote share for party j .

⁶ As discussed in Laver and Shepsle (1996; see also Diermeier & Merlo, 2004), the rules for selecting the formateur vary widely between countries. Some countries, including Greece, pick the largest party; in others such as Britain and Ireland, the party of the outgoing Prime Minister is chosen; whereas in still other countries – notably Italy – parties' policy positions may matter in the sense that the head of state talks to all the parties first, and then picks the one that seems most likely to succeed in forming a governing coalition (Laver & Shepsle, 1996: 53). While it is difficult to empirically evaluate how well the proportionality assumption captures formateur selection in individual countries (due to small sample sizes), in Diermeier and Merlo's (2004: p. 4) study of formateur selections in eleven Western European democracies the authors conclude that in eight of these cases (those of Belgium, Finland, France, Germany, Iceland, Ireland, the Netherlands, and Norway) they cannot reject the proportionality hypothesis.

4 The unified parliament-mean and dominant-party model

The specifications given in Equations (1)–(2) are two nodes of a continuum of utility specifications. Specifications on this continuum represent outcomes intermediate to those of parliamentary-mean and dominant-party expectations. In reality, parties and political elites neither expect that government policy outputs or even control over a single policy portfolio will go entirely to one party or coalition, nor do they expect that a pure compromise among all parties will be achieved on every issue. In this regard, politicians' expectations over the relative policy influence of parliamentary players depend critically upon country-specific power-sharing arrangements. Factors that disperse power among parliamentary parties (as in the parliamentary-mean model) include constitutionally-required supermajorities for important legislation; bicameral legislatures; provisions for opposition parties' participation on important legislative committees; and informal agreements that important bureaucratic positions will "rotate" among representatives from different parties (see Lijphart, 1984, 1999; Laver & Hunt, 1992). Factors that promote policy dominance by a single party or by the governing coalition (as in the dominant-party model) include restrictive legislative procedures (see Huber, 1996), unicameral legislatures, and centralized government (as opposed to federal systems).

Of course – in many polities – policy on each issue is primarily a compromise among the policy positions of the parties making up the government or cabinet. In turn, relative influence of governmental parties on the nature of such compromises is related to their seat shares. We term this the *cabinet-mean* expectation of governmental outcomes. The cabinet-mean model dates back to Gamson (1961), and has been widely used in subsequent empirical and theoretical work (Browne & Franklin, 1973; Morelli, 1999; Huber & Powell, 1994; Powell, 2000). Warwick (2001) finds the cabinet mean to be the strongest predictor of government outputs in his empirical analyses of government policy programmes in ten democracies.

Because the makeup of the governing coalition is not known in advance of the election (even if the composition of prospective coalitions can be surmised, the identity of the winning coalition may not be known), it is difficult to incorporate the cabinet mean into the formal model. We note, however, that the concept of weighting only the positions of parties in government is intermediate between weighting all members of parliament (as in the parliamentary-mean model) and weighting only a single party (as in the dominant-party model). In particular, a cabinet mean – being a weighted average of the positions of two or more parties – is likely to be less extreme than the position of the most extreme parties. Furthermore, just as the identity of the party that becomes the dominant-party is unknown during the election campaign, the identity of the coalition that eventually forms a government and makes policy is probabilistic at election time.⁷ For these reasons it seems plausible that the "unified" model that we present below – one that combines aspects of the parliamentary mean and dominant-party models – captures parties' strategic considerations in situations where the cabinet mean affects policy outputs.

⁷ Given that we specify that the identity of the dominant-party is not known in advance of the election, readers may wonder why we cannot similarly incorporate uncertainty over the identity of the governing coalition into the model. The crucial distinction is that while it appears plausible to assume that a party's probability of becoming dominant in parliament is proportional to its seat share, it is *not* plausible to assume that a proto-coalition's probability of forming the government is proportional to its seat share. For instance if a coalition of right-wing parties is projected to jointly control 45% of the seats in parliament while the rival left-wing proto-coalition is expected to control 55% of the seats, then the right-wing proto-coalition's probability of forming the government is almost surely substantially lower than 0.45. This discrepancy greatly complicates efforts to incorporate parties' projections about governing coalitions into our model.

To define policy-seeking utility to encompass both pure parliamentary-mean and dominant-party as well as intermediate expectations, we specify a unified model under which the utility of party k for the outcome is given by

$$U_k = - \sum_{j=1}^K |P_k - [\alpha s_j + (1 - \alpha)\bar{s}]|EV_j, \quad (3)$$

where α represents the relative policy influence of the dominant-party effect and $(1 - \alpha)$ the relative influence of the parliamentary mean, respectively. If $\alpha = 1$, $U_k = - \sum_{j=1}^K |P_k - s_j|EV_j$, i.e., utility is specified according to dominant-party expectations. On the other hand, if $\alpha = 0$, then $U_k = - \sum_{j=1}^K |P_k - \bar{s}|EV_j = -|P_k - \bar{s}|$, i.e., U_k specifies utility according to parliamentary-mean expectations.

5 Previous research on party strategies and government policy outputs

Before analyzing party strategies for the alternative models of policy-making introduced above, we ask the question: What does previous research imply about the effects of alternative institutions upon parties' policy strategies and government policy outputs? With respect to policy outputs, the answer to this question is clear: previous empirical studies by Powell (2000) and by McDonald, Mendes, and Budge (2004; see also McDonald & Budge, 2006) – both of which evaluate policy-making in over twenty democracies – conclude that institutions that promote power-sharing, as in the parliamentary-mean model, promote more moderate policy outputs than do institutions that concentrate power in the hands of a single party or in the hands of the government, as in the dominant-party model. The logic that supports these empirical findings, which has been developed extensively by Lijphart (1984, 1999; see also Powell, 2000), is equally clear: namely, that by its nature political compromise between parties generates moderate policy outputs. We note, however, that Lijphart, Powell and others do not explicitly consider the possibility that consensual institutions could motivate parties to stake out extreme policy positions – perhaps in an effort to “pull” the eventual compromise policy in the party's preferred direction. To the extent that this is the case, it is not obvious that consensual institutions will actually moderate government policy outputs.

Research on how power-sharing affects parties' policy declarations is less well-developed, especially for the policy-seeking parties that we analyze in this paper. To our knowledge the only previous research that is directly relevant to this issue is by Schofield (1993, 2005), who concludes that institutional arrangements that promote single-party governments – as in the dominant-party model – motivate policy-seeking parties to moderate their positions, but that institutions that promote power-sharing and coalition governments – as in the parliamentary-mean model – provide mixed incentives, i.e. they motivate some parties to moderate their positions but may motivate other parties to stake out relatively extreme positions in order to influence coalition bargaining in their favor (2005, p. 34).

In addition to Schofield's work on policy-seeking parties, Dow (2001) presents intuitive arguments on strategic incentives for *office-seeking* parties, namely that institutional arrangements that promote single-party governments – which are compatible with the dominant-party model – motivate office-seeking parties to maximize their vote shares (since the largest party is likely to be the governing party), thereby promoting policy moderation.

In addition to the party-centered studies discussed above, research on individual-level voting by Kedar (2005) presents arguments that – if applied to parties – suggest that consensual

institutions may motivate policy-seeking parties to adopt more extreme policy positions (see also Hinich, Henning, Shikano, 2004, & Grofman, 1985). Kedar argues that voters in consensual systems – unlike voters in majoritarian systems – may strategically support parties whose announced positions are more extreme than the voters’ sincere preferences over government policy outputs, because voters project that any single party’s policy influence will be diluted by institutional power-sharing arrangements. Although Kedar’s argument concerns voters, her intuitive logic may translate to parties: namely, policy-seeking parties in consensual systems may strategically stake out extreme policy positions, reasoning that these extreme “bargaining positions” at the election stage will help them pull policy outputs in their preferred policy direction, during post-election bargaining and compromise. We expand this argument below, distinguishing between the roles that voters and parties play in this effect.

6 Nash equilibria for the dominant-party and parliamentary mean models

As discussed above the dominant-party and parliamentary-mean assumptions represent dramatically different specifications for government policy outputs. One might suspect that these alternative models would therefore present policy-seeking parties with different sets of strategic incentives; and, in fact, the previous research on this topic summarized above suggests that most political scientists share this intuition (even if they disagree about what these incentives are!).

We assume throughout that voter utilities decline with distance from the voter’s ideal point. The motivations that voters employ when they choose parties may, however, be expressive or instrumental. Under *expressive* motivations, voters express their true or sincere preferences, which we operationalize by assuming that each voter chooses that party whose declared policy position is closest to the ideal point of the voter. Alternatively, a voter may have an *instrumental* motivation, under which she looks ahead to the policy that may be implemented and makes her vote choice in order to maximize (the expected value of) the proximity of the implemented policy to her preferred policy location. Because the declared policy of the party thus chosen may not be closest to the voter’s position, instrumental motivations may lead voters to support different parties than they would if they voted expressively.

6.1 Equilibrium conditions under expressive voter motivations

Under the assumption of expressive motivations for voters, for either the dominant-party or the parliamentary-mean model, we describe explicitly a necessary condition for a Nash equilibrium,⁸ given an assumption about the location of parties’ preferred positions. In a Web Appendix* (see: <http://course.wilkes.edu/merrill/>), we also provide a sufficient condition for existence of a local equilibrium, given certain assumptions (see Theorem W1). Although these assumptions are quite restrictive, numerical calculation suggests that such equilibria exist under a much broader range of conditions than those satisfied by the theorems. The necessary condition, which we present here, is intended to provide insight into the relation between the voter distribution and the locations of parties at equilibrium.

⁸ A Nash equilibrium is a configuration of strategies (s_1, s_2, \dots, s_K) such that no party can increase its utility by unilaterally changing its position.

*Electronic Supplementary Material Supplementary material is available for this article at <http://dx.doi.org/10.1007/s11127-006-9123-z>.

Denote the party positions by $s_1 \leq s_2 \leq \dots \leq s_K$ and the midpoints between the party positions by $m_j = (s_j + s_{j+1})/2$ for $j = 1, \dots, K - 1$.⁹ Let f and F be the voter density function and the voter cumulative distribution function, respectively. The following theorem indicates a necessary condition for a Nash equilibrium.

Theorem 1. *Suppose that parties have linear policy losses, that voters have expressive motivations, and that a Nash equilibrium occurs at a configuration of strategies (s_1, s_2, \dots, s_K) and assume $P_1 \leq P_2 \leq \dots \leq P_K$ and $s_1 \leq s_2 \leq \dots \leq s_K$. For the parliamentary-mean model,*

$$\frac{\partial U_k}{\partial s_k} = f(m_{k-1})(s_k - m_{k-1}) + f(m_k)(m_k - s_k) - [F(m_k) - F(m_{k-1})] = 0, \tag{4}$$

holds for all $k, k = 1, \dots, K$. For the dominant-party model, the same equality holds for all $k, k = 1, \dots, K$ provided that $P_1 \leq s_1, P_K \geq s_K$, and $P_k \leq s_{k-1}$ or $P_k \geq s_{k+1}$ for $1 < k < K$.

Proof: See Appendix A.¹⁰ □

Note that satisfaction of Equation (4) above is geometrically represented by the fact that areas A_1 and B_1 are equal, areas A_2 and B_2 are equal, etc, in Figure 2.¹¹ At equilibrium, this geometric condition implies that party 2 is closer to party 1 than to party 3 if the curve is steeper in the region between party 1 and party 2 than between party 2 and party 3. We will discuss the implications of this observation later.

6.2 Optimal party strategies under instrumental voter motivations

If voters are instrumentally motivated, under the parliamentary-mean model, any voter located to the left of the parliamentary mean \bar{s} can move that mean in the direction of her ideal point by the greatest amount by voting for the most extreme leftwing party; likewise voters to the right of the mean are motivated to vote for the most rightwing party.¹² In the (unlikely) event that all voters voted instrumentally, only the leftmost party and the rightmost party would receive votes. It follows that the leftmost party has an incentive to move left; the rightmost party has an incentive to move right.¹³ In real politics, however, these assumptions are much

⁹ We further denote by m_0 and m_K the left and right hand end points (which may be infinite), respectively, of the voter distribution. We also assume that $f(m_0), f(m_K)$, and products of which they are factors are zero.

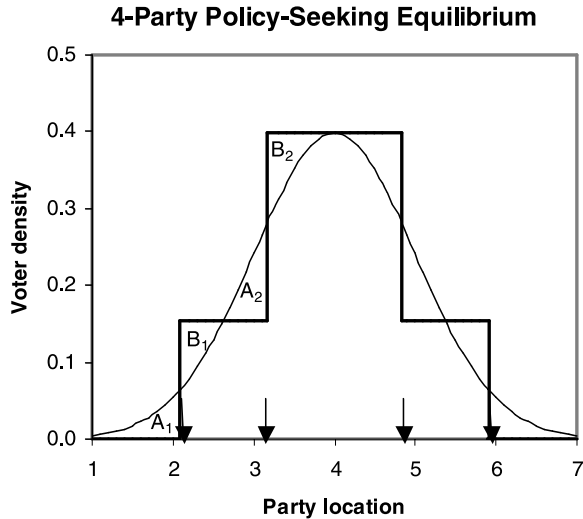
¹⁰ For a proof that, for a special case, party utilities and Nash equilibria are equivalent over models, see Theorem W2, in the Web Appendix (<http://course.wilkes.edu/merrill/>). Under quadratic loss instead of linear loss utility, we also prove (Theorem W3) in the Web Appendix that the most extreme parties are motivated to take more extreme stands under the parliamentary-mean model than under the dominant-party model.

¹¹ To see this, first note that Equation (4) is equivalent to the statement that, for $k = 1, \dots, K$, the sum of the areas of the rectangles with bases $[m_{k-1}, s_k]$ and $[s_k, m_k]$ and heights $f(m_{k-1})$ and $f(m_k)$, respectively, is equal to the area under the probability density function f between m_{k-1} and m_k (m_0 and m_K are the end-points of the voter distribution). Geometrically, the latter statement is equivalent to equality between area A_k and area B_k for $k = 1, \dots, K$.

¹² The only exception would be a voter whose position would be so close to the mean without her ballot that her ballot alone would cause the parliamentary mean to leapfrog her position. We consider such possibilities negligible.

¹³ To see this, recall that m_0 and m_K are the endpoints of the voter distribution (m_0 may be $-\infty$ and m_K may be $+\infty$) and F is the cumulative voter distribution function. Because each voter's choice of the leftmost

Fig. 2 Policy-seeking equilibrium for four parties (Preferred positions for parties are 2, 2, 6, and 6, respectively. Arrows indicate locations of parties at equilibrium; the locations are such that areas $A_1 = B_1$, $A_2 = B_2$, etc. The equilibria depicted apply to the dominant-party and parliamentary-mean models as well as all intermediate models)



too strong for a variety of reasons. Not all voters vote instrumentally and those that do are likely to have expressive motivations as well; indeed, Kedar (2005) – who is one of the most visible proponents of the argument that real world voters instrumentally support extreme parties in order to influence government policy outputs – concludes that these voters are also significantly influenced by expressive motivations. Parties, furthermore, cannot credibly espouse arbitrarily extreme positions as bargaining chips. Overall, however, instrumental voter motivations exert a centrifugal force on party declarations. We return to these concerns later.

On the other hand, under a dominant-party model – for either instrumental or expressive voter motivations – each voter maximizes her utility by voting for the nearest party (for a voter located at v under, say, linear-loss utility, that utility is given by $-\sum_j EV_j |v - s_j|$). Thus, for the dominant-party model, party utilities – and hence equilibrium analyses – are identical whether voter utilities are expressive or instrumental. As suggested by our theoretical result and supported by numerical calculations below, these equilibrium strategies are dispersed but not arbitrarily divergent.

In toto, we infer that – under the assumption of instrumental voter motivations – the parliamentary-mean model provides incentives for parties to diverge substantially more than does the dominant-party model. The dynamics are analogous to the effects of alternative arbitration schemes for dispute settlement, but on a multi-player level (Brams & Merrill 1983, 1991). Like conventional arbitration (under which the arbitrator is free to select a compromise outcome that is between the offers proposed by the two players), the parliamentary-mean

party (located, say, at L) or the rightmost party (located, say, at R) depends on the location of \bar{s} , which in turn depends on the proportion of voters choosing L or R , voters will be in equilibrium only if \bar{s} satisfies the equation $\bar{s} = F(\bar{s})L + (1 - F(\bar{s}))R$. But because the function $g(x) = F(x)L + (1 - F(x))R$ is a continuous and non-increasing function of x , and because $g(m_0) = R > m_0$, and $g(m_K) = L < m_K$, then by the intermediate value theorem, there always exists a unique value \bar{x} in the interval $[m_0, m_K]$ for which $g(\bar{x}) = \bar{x}$, i.e., $\bar{x} = F(\bar{x})L + (1 - F(\bar{x}))R$. But this latter equality implies that this \bar{x} is the parliamentary mean \bar{s} . Implicit differentiation then shows that $\frac{\partial \bar{s}}{\partial L} = \frac{F(\bar{s})}{f(\bar{s})(R-L)+1}$, an expression that is never negative, so that \bar{s} increases as L does. Because the leftmost party prefers that \bar{s} decrease (at least as long as \bar{s} is to the right of L 's preferred position), it moves left; likewise, the rightmost party moves right.

model encourages the players – or at least the two most extreme players – to each reach for the moon in the hopes that the arbitrator (parliament) will compromise at a position shaded in its respective direction. Alternatively, like final offer arbitration (under which the arbitrator must choose a specific offer proposed by one of the players with no option to compromise), the dominant-party model maintains a tradeoff for each party between declaring a more extreme but desired position on the one hand and a more moderate position attractive to the arbitrator (electorate) on the other.

7 Numerical computation of optimal party positions

While the theoretical result reported above is suggestive, it does not prove that there is a *general* tendency for policy-seeking parties' equilibrium strategies to be at least as dispersed under the parliamentary-mean model as they are under the dominant-party model. Here we supplement our theoretical result by reporting computations of policy-seeking equilibria for alternative scenarios that vary the mix of dominant-party and parliamentary-mean expectations, the parties' preferred positions, and whether voter motivations are expressive or instrumental.

7.1 The distribution of parties' policy strategies for expressive voter motivations

Table 1 reports computed Nash equilibrium configurations for four-party elections under the assumption of expressive voter motivations. We assume that the voter distribution is normal with mean 4 and a standard deviation of 1 on the conventional 1–7 policy scale and that the parties have linear policy losses. For each equilibrium configuration, we also report the standard deviation of the parties' positions.¹⁴

If, as in example A in Table 1, the preferred positions are the locations $P_1 = 2$, $P_2 = 2$, $P_3 = 6$, and $P_4 = 6$ along this scale, the equilibrium configuration finds the parties presenting a dispersed set of strategies satisfying the conditions of Theorem 1. Party 1 locates at approximately the 3rd percentile of the voter distribution while parties 2–4 locate at the 20th, 80th, and 97th percentiles, respectively.¹⁵ The standard deviation of the parties' equilibrium strategies, 1.48, substantially exceeds the standard deviation of the voter distribution, indicating that the parties' positions are more dispersed than are the policy preferences of the electorate they are competing to represent. In addition, we note that at equilibrium each party's preferred position is less centrist than its own strategy and those of the adjacent parties along the policy scale (the conditions specified in Theorem 1). Given that empirical research

¹⁴ Expected government policy outputs – reported in Table 1 – will be discussed later. Computed equilibria for elections with any number of candidates between three and ten and for quadratic-loss as well as linear-loss utility for parties are reported in Tables W1 and W2 on the Web Appendix (<http://course.wilkes.edu/merrill/>). The results are qualitatively similar to those presented here for four parties, but with the dispersion of optimal strategies generally increasing with the number of parties. Party positions at equilibrium are computed using an iterated approximation procedure, which successively alters one party strategy at a time to optimize that party's utility until no further changes in party locations are needed, with precision to two decimal places. Convergence to a unique equilibrium location was obtained in each case illustrated in Table 1 as well as Tables W1 and W2 (except for leapfrogging or for interchanging the strategies of parties with identical preferred positions).

¹⁵ We note that for this equilibrium configuration the strategies of parties P_1 and P_2 , as well as the strategies of P_3 and P_4 , are interchangeable. Although a configuration involving parties having identical preferred positions is unlikely, we include such a scenario in Table 1 to emphasize that the dispersion of optimal strategies at equilibrium does not depend on separateness of the preferred positions of the parties.

Table 1 Nash equilibria for policy-seeking parties in a four-party election, for *pure and unified* models of policy expectations

	A. Preferred positions: ($P_1 = 2, P_2 = 2, P_3 = 6, P_4 = 6$)			B. Preferred positions: ($P_1 = 1, P_2 = 3, P_3 = 5, P_4 = 7$)		
	Dominant-party model (1)	Parliamentary-mean model (2)	Unified model (3)	Dominant-party model (4)	Parliamentary-mean model (5)	Unified model (6)
Party 1	2.08	2.08	2.08	1.97	2.08	2.08
Party 2	3.17	3.17	3.17	3.00	3.17	3.17
Party 3	4.83	4.83	4.83	5.00	4.83	4.83
Party 4	5.92	5.92	5.92	6.03	5.92	5.92
Standard deviation of party positions	1.48	1.48	1.48	1.60	1.48	1.48
Expected deviation of policy outputs	1.01	0.0	0.51	1.13	0.0	0.51

	C. Preferred positions: ($P_1 = 2, P_2 = 3, P_3 = 5, P_4 = 6$)			D. Preferred positions: ($P_1 = 3, P_2 = 3.5, P_3 = 4, P_4 = 5$)		
	Dominant-party model (1)	Parliamentary-mean model (2)	Unified model (3)	Dominant-party model (4)	Parliamentary-mean model (5)	Unified model (6)
Party 1	2.00	2.08	2.08	3.00	2.08	2.03
Party 2	3.00	3.17	3.17	3.50	3.17	3.03
Party 3	5.00	4.83	4.83	4.00	4.83	4.03
Party 4	6.00	5.92	5.92	5.00	5.92	5.52
Standard deviation of party positions	1.58	1.48	1.48	0.74	1.48	1.29
Expected deviation of policy outputs	1.13	0.0	0.51	0.62	0.0	0.36

Notes: In the table, the unified model assumes a mixing parameter of 0.5, i.e., equal weights for the dominant-party and parliamentary-mean components. The equilibria were computed assuming linear-loss utilities for parties, expressive voter motivations, and a normal voter distribution centered on 4.0 with a standard deviation of 1.0. For the scenario with preferred positions: ($P_1 = 2, P_2 = 2, P_3 = 6, P_4 = 6$), the equilibrium positions of parties 1 and 2, as well as those of parties 3 and 4, are interchangeable. *Expected deviation of policy outputs* represents the expected distance between government policy outputs and the median voter position, as given by Equation (5).

finds that political elites typically have more extreme policy preferences than do rank-and-file voters (see Dalton, 1985; Iversen, 1994), we speculate that this condition will frequently be satisfied in real world elections.

In general, the results reported in Table 1 illustrate the conclusion on party strategies demonstrated by Theorem 1. In example A, the equilibria are identical for all pure and unified (mixed) models presented (see Table 1). For $P_1 = 1, P_2 = 3, P_3 = 5, P_4 = 7$ (example B) or for $P_1 = 2, P_2 = 3, P_3 = 5, P_4 = 6$ (example C), the equilibria are virtually identical across models, even though the conditions of Theorem 1 are not entirely satisfied.¹⁶

Furthermore, for the parliamentary-mean model, the configuration of Nash equilibrium strategies does not depend on the parties' preferred positions, as long as \bar{s} does not cross any of the P_k .¹⁷ Under the dominant-party model, however, equilibrium strategies do depend on the P_k , in a complicated fashion, but numerical calculations suggest that this variation in strategies is relatively small, except that a party's optimal strategy is never more extreme than its preferred position, so that parties that prefer a policy close to the center of the voter distribution locate close to that center, as illustrated by example D in Table 1. Thus, overall, the parliamentary-mean and dominant-party models yield fairly similar strategies when the parties' preferred positions are well-dispersed, but the parliamentary-mean model (as well as a unified model with a significant parliamentary-mean component) motivates more extreme strategies than the dominant-party model for parties preferring more centrist positions.¹⁸ Further numerical computation (not shown) assuming that a party's chances of becoming dominant depend on proximity to the center of the voter distribution as well as vote share suggest that peripheral parties are drawn in, thereby reducing the variance of the party distribution.

Furthermore, we note that in all scenarios with voters normally distributed – including those for which the parties' preferred positions are evenly distributed – the equilibrium positions for parties 1 and 2 (or for parties 3 and 4) are closer together than are the corresponding positions for parties 2 and 3. Thus, the two parties preferring left-wing positions are grouped, as are the two preferring right-wing positions. A similar argument applies to other voter distributions that are flatter in the middle than on the flanks. See Appendix B for the effect of distribution shape on this expectation. We may expect, however, that the distribution of party declarations at equilibrium is dispersed, regardless of whether the parties' preferred positions are themselves separate. Furthermore – insofar as the voter distribution resembles a normal curve or other curve that is flatter in the middle than on the flanks and no party holds a centrist position – the distribution of partisan strategies will be split into two groups: one on the left and one on the right.

7.2 Party declarations for instrumental or mixed voter motivations

As we have seen – under the parliamentary-mean model – if all voters implement instrumental motivations, their vote choices exert a strong centrifugal force on party declarations. If, more plausibly, voters have mixtures of expressive and instrumental motivations – as Kedar's (2005) empirical analyses imply – we can expect this centrifugal force to be less pronounced, so that party declarations are only moderately more extreme than under purely

¹⁶ Each party's preferred position is not more extreme than the positions of the adjacent party's strategies.

¹⁷ For the parliamentary-mean model, this follows because $\frac{\partial U_k}{\partial s_k}$ does not depend on P_k unless P_k and \bar{s} cross.

¹⁸ Note that in these examples, the variance of optimal strategies is either the same or similar over models or is significantly greater for models with a parliamentary-mean component.

expressive motivations.¹⁹ But our numerical calculations above suggest that, for expressive motivations, party declarations should be similar under both the parliamentary-mean and dominant-party models. Since party strategies under the dominant-party model do not depend on voter motivations, we conclude that under mixed voter motivations, party declarations will be moderately more extreme under the parliamentary-mean model than under the dominant-party model. These conclusions are summarized in the following table:

	Parliamentary-mean model	Dominant-party model
Expressive motivations	Moderate dispersion	Moderate dispersion
Mixed motivations	Sharp dispersion	Moderate dispersion
Instrumental motivations	Extreme dispersion	Moderate dispersion

7.3 Numerical results on government policy outputs

For each equilibrium configuration we also computed the expected deviation between government policy outputs and the median voter position – which we labeled *expected deviation of policy outputs*. This quantity is defined as the mean of the distances between each possible government policy output and the median voter position (4.0), weighted by the likelihood of each policy outcome:

Expected deviation of policy outputs

$$U_k = \sum_{j=1}^K |\alpha s_j + (1 - \alpha)\bar{s} - 4|EV_j. \quad (5)$$

This measure is important because, as noted above, Lijphart (1999) argues that an advantage of consensual political systems – which are compatible with the parliamentary-mean model – is that they promote moderate policy outputs. In addition, previous empirical studies by Powell (2000) and McDonald, Mendes, and Budge (2004; see also McDonald & Budge, 2006) assess the quality of representation in real world political systems by estimating the deviation between government policy outputs and the median voter's policy preferences.

The results reported in Table 1 show that expected congruence between the median voter position and government policy outputs is substantially tighter for the parliamentary-mean model than for the dominant-party model. These computations thereby suggest that institutions that promote power-sharing and political compromise will moderate policy outcomes.

In toto, our computations on party equilibria support our theoretical results, that when citizens vote expressively, expectations of parliament-wide power-sharing at the pre-election stage do not systematically motivate policy-seeking parties to present more extreme positions than they would present if they expected a single party or a governing coalition to dominate the policy-making process. By contrast, expectations of power-sharing motivate instrumental voters to support more extreme parties, which in turn motivates political parties to present more extreme policy positions.

¹⁹ We do not attempt to model voter utilities formally under mixed motivations, because the utilities for expressive and instrumental motivations are not on the same scale.

8 Conclusion

Policy outputs following an election using proportional representation may follow a process in which a single party is preeminent. Alternatively, policy may combine the interests of the members of a governing coalition of parties, or, at the extreme, of all parties in parliament. We have labeled the two poles of these alternatives the dominant-party model and the parliamentary-mean model. Assuming that parties are policy-seeking, we have described conditions that obtain for a configuration of parties' optimal policy strategies (a Nash equilibrium), and we have explored the nature of optimal strategy configurations for the two pure models as well as for unified models of which the pure models are components. To our knowledge, this represents the first spatial model that presents such equilibrium analyses for multiparty elections with policy-seeking parties.

Unlike the typically convergent equilibria that occur in two-party contests under office-seeking motivations, multiparty equilibrium configurations under policy-seeking motivations are typically dispersed, but – so long as voter motivations are expressive – not as dispersed as the sincere policy preferences of the parties themselves. Thus, in this sense, parties and voters have no reason not to take these positions seriously.

Our major finding is that – among various models of government formation comprising dominant-party and parliamentary-mean assumptions – the degree of dispersion of optimal policy configurations is either comparable or in some cases greater under power-sharing institutions. In particular, insofar as voters are motivated by instrumental considerations, parliamentary-mean decision making may motivate parties to stake out substantially more divergent positions than does the dominant-party model.

For nearly every equilibrium that we computed – regardless of the mixture of dominant-party and parliamentary-mean expectations, or the parties' preferred positions – the equilibrium positions are significantly dispersed in the policy space, suggesting that policy-motivated parties will present voters with a wide range of policy options.²⁰ This conclusion obtains despite the fact that we assume no uncertainty about the location of the voter distribution; if we were to incorporate such uncertainty into the model, we would expect even greater variance in the party configuration at equilibrium.

Our theoretical results on party strategies are relevant to recent cross-national studies by Budge and McDonald (2006) and Ezrow (2005), which conclude that the dispersion of parties' policy declarations observed across different party systems is not consistently linked to the power-sharing arrangements in these systems. Intuitively, these authors' empirical findings appear surprising; however they are precisely what our theoretical results imply. Furthermore, our analyses supply the intuition for these empirical findings: namely, that regardless of institutional arrangements, policy-seeking parties must balance their desires to declare policies that reflect their sincere preferences – but which may attract only modest electoral support – against the strategy of moderating their declared positions in order to increase support and hence increase their policy influence.

Our findings on policy-seeking parties' strategic incentives under alternative power-sharing institutions are important for another reason: namely, they are directly relevant to the long-standing debate over the merits of *consensual* democracies that feature widespread power-sharing – a debate that is especially relevant given the current interest in the appropriate institutions for societies as diverse as Iraq and Northern Ireland. A central argument advanced

²⁰ The only exceptions to this rule occur under the dominant-party model for configurations in which all parties' preferred positions are closely bunched near the center of the voter distribution – scenarios unlikely to occur in the real world.

by the advocates of consensual institutions is that they promote moderate policy outputs that are acceptable to a wide spectrum of groups in society (see Lijphart, 1984, 1999; Powell, 2000). However, if consensual institutions motivate parties to propose more extreme policies at the pre-election stage (as several of the scholars cited earlier have argued), then it would be far from obvious that resulting government policy outputs at the post-election bargaining stage would in fact be moderate. Our analysis suggests that insofar as voters focus on outcomes, power-sharing institutions do indeed create a centrifugal force among policy declarations at the pre-election stage. Our results, however, illuminate the empirical, cross-national, findings reported by Powell (2000) and by McDonald, Mendes, and Budge (2004; see also McDonald & Budge, 2006), that government policy outputs more closely reflect the median voter's policy preferences in consensual political systems than they do in majoritarian systems.

We emphasize that our spatial model assumes a one-dimensional policy continuum and that influence in parliament is proportional to vote share. The utility functions, however, for both the parliamentary-mean and dominant-party models could easily be extended to multiple policy dimensions and – although the party vote shares would be more difficult to compute – we speculate that the conclusions concerning the relative divergence of party positions would be substantively similar. Likewise, if parties located nearer the center of the voter distribution are disproportionately likely to become dominant, then under the dominant-party model the variance of the parties' optimal locations is reduced so that our conclusion that divergence may be less than under the parliamentary-mean model would be even more compelling. These and other alternative assumptions – such as a parliamentary-median model – may be investigated in future research.

Furthermore, our model focuses on just one of the many institutional variables that affect power-sharing in real world democracies. Other institutional variables that affect power-sharing include: the voting system, in particular PR versus plurality (see footnote 1); the presence or absence of a written constitution; the degree to which the political system is federalist (decentralized) as opposed to unitary (centralized); the number of parties; provisions for interest group representation (corporatist versus pluralist); provisions for judicial review of legislation; unicameral versus bicameral legislatures; presidential versus parliamentary democracy; the degree of independence of the central bank.²¹ Given this extensive list, we make no claims that our results provide a comprehensive theoretical analysis of the effects of power-sharing institutions. Nevertheless, the policy primacy of the dominant-party vis-à-vis the cabinet and the parliament is an important power-sharing variable, and we believe that our findings shed light on how this variable influences party strategies and the resulting government policy outputs. We therefore see our approach as providing a useful first step in the quest to bring spatial modeling techniques to bear on issues of institutional design in real world political systems.

Appendix A: Proof of Theorem 1

Theorem 1. *Suppose that parties have linear policy losses, that voters have expressive motivations, and that a Nash equilibrium occurs at a configuration of strategies (s_1, s_2, \dots, s_K) and assume $P_1 \leq P_2 \leq \dots \leq P_K$ and $s_1 \leq s_2 \leq \dots \leq s_K$. For the parliamentary-mean model, Equation (4):*

$$\frac{\partial U_k}{\partial s_k} = f(m_{k-1})(s_k - m_{k-1}) + f(m_k)(m_k - s_k) - [F(m_k) - F(m_{k-1})] = 0$$

²¹ See Lijphart (1984, 1999) for a thorough review of the importance of these variables.

holds for all $k, k = 1, \dots, K$. For the dominant-party model, the same equality holds for all $k, k = 1, \dots, K$ provided that $P_1 \leq s_1, P_K \geq s_K$, and $P_k \leq s_{k-1}$ or $P_k \geq s_{k+1}$ for $1 < k < K$.²²

Proof: Under a deterministic policy voting model, for any configuration of party positions with $s_1 \leq s_2 \leq \dots \leq s_K$, each of the “interior” parties s_2, \dots, s_{K-1} receive the votes of voters between the midpoints immediately to the left and right of the party position. Thus, the proportion of the vote received by an interior party j is given by the integral of the voter density function between the points m_{j-1} and m_j , where $m_j = (s_j + s_{j+1})/2$ for $j = 1, \dots, K - 1$ (i.e., the midpoints between the party positions). If we further denote by m_0 the left hand end point of the policy scale and by m_K the right hand end point of the policy scale, the same formula extends to party 1 and party K .

For the parliamentary-mean model, for fixed k ($1 < k < K$), if $P_k \leq \bar{s}$,

$$\begin{aligned} \frac{\partial U_k}{\partial s_k} &= \frac{\partial}{\partial s_k} \{-|P_k - \bar{s}|\} = \frac{\partial}{\partial s_k} \left\{ \sum_{j=1}^K (P_k - s_j) EV_j \right\} \\ &= \frac{\partial}{\partial s_k} \left\{ \sum_{j=k-1}^{k+1} (P_k - s_j) [F(m_j) - F(m_{j-1})] \right\}, \end{aligned} \tag{A.1}$$

where F is the cumulative distribution function for the voter distribution. For the dominant-party model, if $P_k \leq s_{k-1}$,

$$\frac{\partial U_k}{\partial s_k} = \frac{\partial}{\partial s_k} \left\{ - \sum_{j=k-1}^{k+1} |P_k - s_j| EV_j \right\} = \frac{\partial}{\partial s_k} \left\{ \sum_{j=k-1}^{k+1} (P_k - s_j) [F(m_j) - F(m_{j-1})] \right\}, \tag{A.2}$$

so that for either model:

$$\begin{aligned} \frac{\partial U_k}{\partial s_k} &= \frac{\partial}{\partial s_k} \{ (P_k - s_{k-1}) [F(m_{k-1}) - F(m_{k-2})] + (P_k - s_k) [F(m_k) - F(m_{k-1})] \\ &\quad + (P_k - s_{k+1}) [F(m_{k+1}) - F(m_k)] \} \\ &= \frac{\partial}{\partial s_k} \{ F(m_{k-1})(s_k - s_{k-1}) + F(m_k)(s_{k+1} - s_k) \} \\ &= \frac{f(m_{k-1})}{2} (s_k - s_{k-1}) + F(m_{k-1}) + \frac{f(m_k)}{2} (s_{k+1} - s_k) - F(m_k) \\ &= f(m_{k-1})(s_k - m_{k-1}) + f(m_k)(m_k - s_k) - [F(m_k) - F(m_{k-1})] = 0 \end{aligned} \tag{A.3}$$

at a Nash equilibrium. If $P_k > \bar{s}$, the sign change in A.1 has no effect on A.3. If $P_k \geq s_{k+1}$ ($1 < k < K$), similar analyses lead to the same conclusions as above.

Similar arguments show that also $\frac{\partial U_1}{\partial s_1} = (m_1 - s_1)f(m_1) - F(m_1) = 0$ and $\frac{\partial U_K}{\partial s_K} = (s_K - m_{K-1})f(m_{K-1}) - [1 - F(m_{K-1})] = 0$. Thus, Equation (4) is a necessary condition for a Nash equilibrium for $k = 1, \dots, K$. This completes the proof of Theorem 1. \square

²² We thank an anonymous referee for improving the statement and proof of Theorem 1.

Table A1. Nash equilibria for policy-seeking parties in a four-party election, for a *triangular* voter distribution

	A. Preferred positions: ($P_1 = 2, P_2 = 2, P_3 = 6, P_4 = 6$)			B. Preferred positions: ($P_1 = 1, P_2 = 3, P_3 = 5, P_4 = 7$)		
	Dominant-party model (1)	Parliamentary-mean model (2)	Unified model (3)	Dominant-party model (4)	Parliamentary-mean model (5)	Unified model (6)
Party 1	2.00	1.75	1.75	1.67	1.75	1.68
Party 2	3.33	3.25	3.25	3.00	3.25	3.03
Party 3	4.67	4.75	4.75	5.00	4.75	4.97
Party 4	6.00	6.25	6.25	6.33	6.25	6.32
Standard deviation of party positions	1.49	1.68	1.68	1.80	1.68	1.78
Expected deviation of policy outputs	1.08	0.0	0.56	1.26	0.0	0.62

Notes. In the table, the unified model assumes a mixing parameter of 0.5, i.e., equal weights for the dominant-party and parliamentary-mean components. The equilibria were computed assuming linear-loss utilities for parties, expressive voter motivations, and a symmetric triangular distribution supported on the interval [1, 7]

Appendix B: Alternative voter distributions

The bunching of party strategies observed earlier for a normally distributed electorate disappears or is reduced if the voters follow a distribution with uniformly-sloped sides, i.e., a triangular distribution. In fact, as indicated in Table A1, for a triangular distribution, parties' optimal strategies for preferred positions at (2, 2, 6, 6) are exactly evenly distributed; whereas, for a normal distribution (see Table 1), the gap between the two middle parties 2 and 3 is about 50 percent greater than that between parties 1 and 2 or between parties 3 and 4. The reason for this can be seen by drawing a figure comparable to Figure 2 but with a triangular distribution. For preferred positions at (1, 3, 5, 7), the optimal strategies at equilibrium vary somewhat over models, but again the "gap-in-the-middle" configuration is consistently more pronounced when voters follow a normal rather than a triangular distribution.²³

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²³ Empirical evidence does not appear to clearly favor either a triangular or a normal shape for the distribution of voters. See, for example, Adams, Merrill, and Grofman (2005) for evidence concerning voter distributions in France (p. 76), Norway (p. 97), and Britain (p. 155).

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